International Research Journal of Education and Technology

Peer Reviewed Journal

ISSN 2581-7795

On the Positive Pell Equation $y^2 = 3x^2 + \alpha^2 + 10\alpha - 2$

V.Anbuvalli[1], K.Kaviya[2]

[1]Assistant professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.

[2]M.Sc Scholar, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu,India.

ABSTRACT:

A binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is non-square positive integer has been study by various mathematicians for it non-trivial integral solutions when D takes different integral values. The binary quadratic Diophantine equation represented by the positive pellian $y^2 = 3x^2 + \alpha^2 + 10\alpha - 2$ is analysed for its non-zero distinct solutions. A few interesting relations among the solutions are given. Further,employing the solutions of the above hyperbola, the solutions of other choices of hyperbolas and parabolas are obtained.

KEYWORDS*:*

Binary quadratic, Hyperbola, Parabola, Pell equation, Integral solutions.

2010 Mathematics subject classification: 11D09

INTRODUCTION*:*

A binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is non-square positive integer has been study by various mathematicians for it non-trivial integral solutions when D takes different integral values [1-3]. For an extensive review of various problems, one may refer [4-13]. In this communication, yet another interesting hyperbola given by $y^2 = 3x^2 + \alpha^2 + 10\alpha - 2$ is considered and infinitely many integer solutions are obtained. A few interesting properties among the solutions are obtained. Further,employing the solutions of the above hyperbola, we have obtained solutions of other choices of hyperbola and parabola.

METHOD OF ANALYSIS*:*

The Positive Pell equation representing hyperbola under consideration is

$$y^2 = 3x^2 + \alpha^2 + 10\alpha - 2 \qquad (1)$$

Whose smallest positive integer solution is

$$x_0 = 3, y_0 = \alpha + 5$$

To obtain the other solutions of (1), consider the Pell equation

$$y^2 = 3x^2 + 1$$

Whose general solution is given by

$$\tilde{x}_n = \frac{1}{2\sqrt{3}} g_n ; \tilde{y}_n = \frac{1}{2} f_n$$

Where

$$f_n = (2+\sqrt{3})^{n+1} + (2-\sqrt{3})^{n+1}$$
$$g_n = (2+\sqrt{3})^{n+1} - (2-\sqrt{3})^{n+1}$$

Applying Brahmagupta lemma between $(x_0, y_0) \& (\tilde{x}_n, \tilde{y}_n)$ the other integer solution of (1) are given by

$$x_{n+1} = \frac{9 f_n}{6} + \frac{(\alpha+5)\sqrt{3} g_n}{6}$$

$$y_{n+1} = \frac{(\alpha+5) f_n}{2} + \frac{3\sqrt{3} g_n}{2}$$

The recurrence relations satisfied by x and y are given by

$$x_{n+1} - 4x_{n+2} + x_{n+3} = 0$$

$$y_{n+1} - 4y_{n+2} + y_{n+3} = 0$$

A few numerical examples are given in the following table: 1

Table: 1 Numerical values

n	x_n	y_n
0	3	$\alpha+5$
1	$\alpha+11$	$2\alpha+19$
2	$4\alpha+41$	$7\alpha+71$
3	$15\alpha+153$	$26\alpha+265$
4	$56\alpha+571$	$97\alpha+989$

From the above table we observe some interesting properties among the solutions which are presented below:

1.Relations between solutions

- $x_{n+1} - 4x_{n+2} + x_{n+3} = 0$
- $2x_{n+1} - x_{n+2} + y_{n+1} = 0$
- $x_{n+1} - 2x_{n+2} + y_{n+2} = 0$
- $2x_{n+1} - 7x_{n+2} + y_{n+3} = 0$
- $7x_{n+1} - x_{n+3} + 4y_{n+1} = 0$
- $x_{n+1} - x_{n+3} + 2y_{n+2} = 0$
- $x_{n+1} - 7x_{n+3} + 4y_{n+3} = 0$

International Research Journal of Education and Technology

Peer Reviewed Journal

ISSN 2581-7795

- $3x_{n+1} + 2y_{n+1} - y_{n+2} = 0$
- $12x_{n+1} + 7y_{n+1} - y_{n+3} = 0$
- $3x_{n+1} + 7y_{n+2} - 2y_{n+3} = 0$
- $y_{n+1} + 7x_{n+2} - 2x_{n+3} = 0$
- $y_{n+2} + 2x_{n+2} - x_{n+3} = 0$
- $y_{n+3} + x_{n+2} - 2x_{n+3} = 0$
- $y_{n+1} + 3x_{n+2} - 2y_{n+2} = 0$
- $y_{n+1} + 6x_{n+2} - y_{n+3} = 0$
- $2y_{n+2} + 3x_{n+2} - y_{n+3} = 0$
- $7y_{n+2} - 2y_{n+1} - 3x_{n+3} = 0$
- $7y_{n+3} - y_{n+1} - 12x_{n+3} = 0$
- $2y_{n+3} - y_{n+2} - 3x_{n+3} = 0$
- $y_{n+3} - 4y_{n+2} + y_{n+1} = 0$

2.Each of the following expressions represents a cubical integers

- $\frac{1}{(\alpha^2 + 10\alpha - 2)}[[(2\alpha + 10)x_{3n+4} - (38 + 4\alpha)x_{3n+3}] + 3[(2\alpha + 10)x_{n+2} - (38 + 4\alpha)x_{n+1}]]$
- $\frac{1}{2(\alpha^2 + 10\alpha - 2)}[[(\alpha + 5)x_{3n+5} - (71 + 7\alpha)x_{3n+3}] + 3[(\alpha + 5)x_{n+3} - (71 + 7\alpha)x_{n+1}]]$
- $\frac{1}{(\alpha^2 + 10\alpha - 2)}[[(2\alpha + 10)y_{3n+3} - 18x_{3n+3}] + 3[(2\alpha + 10)y_{n+1} - 18x_{n+1}]]$
- $\frac{1}{(\alpha^2 + 10\alpha - 2)}[[(\alpha + 5)y_{3n+4} - (33 + 3\alpha)x_{3n+3}] + 3[(\alpha + 5)y_{n+2} - (33 + 3\alpha)x_{n+1}]]$
- $\frac{1}{7(\alpha^2 + 10\alpha - 2)}[[(2\alpha + 10)y_{3n+5} - (246 + 24\alpha)x_{3n+3}] + 3[(2\alpha + 10)y_{n+3} - (246 + 24\alpha)x_{n+1}]]$
- $\frac{1}{3(\alpha^2 + 10\alpha - 2)}[[(12\alpha + 114)x_{3n+5} - (426 + 42\alpha)x_{3n+5}] + 3[(12\alpha + 114)x_{n+3} - (426 + 42\alpha)x_{n+2}]]$
- $\frac{1}{2(\alpha^2 + 10\alpha - 2)}[[(4\alpha + 38)y_{3n+3} - (18)x_{3n+4}] + 3[(4\alpha + 38)y_{n+1} - (18)x_{n+2}]]$
- $\frac{1}{(\alpha^2 + 10\alpha - 2)}[[(4\alpha + 38)y_{3n+4} - (6\alpha + 66)x_{3n+4}] + 3[(4\alpha + 38)y_{n+2} - (6\alpha + 66)x_{n+2}]]$
- $\frac{1}{2(\alpha^2 + 10\alpha - 2)}[[(4\alpha + 38)y_{3n+5} - (246 + 24\alpha)x_{3n+4}] + 3[(4\alpha + 38)y_{n+3} - (246 + 24\alpha)x_{n+2}]]$
- $\frac{1}{7(\alpha^2 + 10\alpha - 2)}[[(14\alpha + 142)y_{3n+3} - (18)x_{3n+5}] + 3[(14\alpha + 142)y_{n+1} - (18)x_{n+3}]]$
- $\frac{1}{2(\alpha^2 + 10\alpha - 2)}[[(14\alpha + 142)y_{3n+4} - (6\alpha + 66)x_{3n+5}] + 3[(14\alpha + 142)y_{n+2} - (6\alpha + 66)x_{n+3}]]$
- $\frac{1}{(\alpha^2 + 10\alpha - 2)}[[(14\alpha + 142)y_{3n+5} - (246 + 24\alpha)x_{3n+5}] + 3[(14\alpha + 142)y_{n+3} - (246 + 24\alpha)x_{n+3}]]$
- $\frac{1}{(\alpha^2 + 10\alpha - 2)}[[(2\alpha + 22)y_{3n+3} - 6y_{3n+4}] + 3[(2\alpha + 22)y_{n+1} - 6y_{n+2}]]$

- $\frac{1}{4(\alpha^2+10\alpha-2)}[[(8\alpha+82)y_{3n+3}-6y_{3n+5}]+3[(8\alpha+82)y_{n+1}-6y_{n+3}]]$
- $\frac{1}{(\alpha^2+10\alpha-2)}[[(8\alpha+82)y_{3n+4}-(22+2\alpha)y_{3n+5}]+3[(8\alpha+82)y_{n+2}-(22+2\alpha)y_{n+3}]]$

3.Each of the following expressions represents a Bi-quadratic Integer

- $\frac{1}{(\alpha^2+10\alpha-2)}[(2\alpha+10)x_{4n+5}-(38+4\alpha)x_{4n+4}+4[(2\alpha+10)x_{2n+3}-(38+4\alpha)x_{2n+2}]]+6$
- $\frac{1}{2(\alpha^2+10\alpha-2)}[(\alpha+5)x_{4n+6}-(71+7\alpha)x_{4n+4}+4[(\alpha+5)x_{2n+4}-(71+7\alpha)x_{2n+2}]]+6$
- $\frac{1}{(\alpha^2+10\alpha-2)}[(2\alpha+10)y_{4n+4}-18x_{4n+4}+4[(2\alpha+10)y_{2n+2}-18x_{2n+2}]]+6$
- $\frac{1}{(\alpha^2+10\alpha-2)}[(\alpha+5)y_{4n+5}-(33+3\alpha)x_{4n+4}+4[(\alpha+5)y_{2n+3}-(33+3\alpha)x_{2n+2}]]+6$
- $\frac{1}{7(\alpha^2+10\alpha-2)}[(2\alpha+10)y_{4n+6}-(246+24\alpha)x_{4n+4}+4[(2\alpha+10)y_{2n+4}-(246+24\alpha)x_{2n+2}]]+6$
- $\frac{1}{3(\alpha^2+10\alpha-2)}[(12\alpha+114)x_{4n+6}-(426+42\alpha)x_{4n+5}+4[(12\alpha+114)x_{2n+4}-(426+42\alpha)x_{2n+3}]]+6$
- $\frac{1}{2(\alpha^2+10\alpha-2)}[(4\alpha+38)y_{4n+4}-18x_{4n+5}+4[(4\alpha+38)y_{2n+2}-18x_{2n+3}]]+6$
- $\frac{1}{(\alpha^2+10\alpha-2)}[(4\alpha+38)y_{4n+5}-(66+6\alpha)x_{4n+5}+4[(4\alpha+38)y_{2n+3}-(66+6\alpha)x_{2n+3}]]+6$
- $\frac{1}{2(\alpha^2+10\alpha-2)}[(4\alpha+38)y_{4n+6}-(246+24\alpha)x_{4n+5}+4[(4\alpha+38)y_{2n+4}-(246+24\alpha)x_{2n+3}]]+6$
- $\frac{1}{7(\alpha^2+10\alpha-2)}[(14\alpha+142)y_{4n+4}-18x_{4n+6}+4[(14\alpha+142)y_{2n+2}-18x_{2n+4}]]+6$
- $\frac{1}{2(\alpha^2+10\alpha-2)}[(14\alpha+142)y_{4n+5}-(66+6\alpha)x_{4n+6}+4[(14\alpha+142)y_{2n+3}-(66+6\alpha)x_{2n+4}]]+6$
- $\frac{1}{(\alpha^2+10\alpha-2)}[(14\alpha+142)y_{4n+6}-(246+24\alpha)x_{4n+6}+4[(14\alpha+142)y_{2n+4}-(2466+24\alpha)x_{2n+4}]]+6$
- $\frac{1}{(\alpha^2+10\alpha-2)}[(2\alpha+22)y_{4n+4}-6y_{4n+5}+4[(2\alpha+22)y_{2n+2}-6y_{2n+3}]]+6$
- $\frac{1}{4(\alpha^2+10\alpha-2)}[(8\alpha+82)y_{4n+4}-6y_{4n+6}+4[(8\alpha+82)y_{2n+2}-6y_{2n+4}]]+6$
- $\frac{1}{(\alpha^2+10\alpha-2)}[(8\alpha+82)y_{4n+5}-(22+2\alpha)y_{4n+6}+4[(8\alpha+82)y_{2n+3}-(22+2\alpha)y_{2n+4}]]+6$

4.Each of the following expressions represents a Quintic Integer

- $$\frac{1}{(\alpha^2+10\alpha-2)}\left[(2\alpha+10)x_{5n+6}-(38+4\alpha)x_{5n+5}+5[(2\alpha+10)x_{5n+6}-(38+4\alpha)x_{5n+5}]+10[(2\alpha+2)x_{n+2}-(38+4\alpha)x_{n+1}]\right]$$
- $$\frac{1}{2(\alpha^2+10\alpha-2)}\left[(\alpha+5)x_{5n+7}-(71+7\alpha)x_{5n+5}+5[(\alpha+5)x_{3n+2}-(71+7\alpha)x_{3n+3}]+10[(\alpha+5)x_{n+3}-(71+7\alpha)x_{n+1}]\right]$$
- $$\frac{1}{(\alpha^2+10\alpha-2)}\left[(2\alpha+10)y_{5n+5}-18x_{5n+5}+5[(2\alpha+10)y_{3n+3}-18x_{3n+3}]+10[(2\alpha+10)y_{n+1}-18x_{n+1}]\right]$$
- $$\frac{1}{(\alpha^2+10\alpha-2)}\left[(\alpha+5)y_{5n+6}-(33+3\alpha)x_{5n+5}+5[(\alpha+5)y_{3n+4}-(33+3\alpha)x_{3n+3}]+10[(\alpha+5)y_{n+2}-(33+3\alpha)x_{n+1}]\right]$$
- $$\frac{1}{7(\alpha^2+10\alpha-2)}\left[(2\alpha+10)y_{5n+7}-(246+24\alpha)x_{5n+5}+5[(2\alpha+10)y_{3n+5}-(246+24\alpha)x_{3n+3}]+10[(2\alpha+10)y_{n+3}-(246+24\alpha)x_{n+1}]\right]$$
- $$\frac{1}{3(\alpha^2+10\alpha-2)}\left[(12\alpha+114)x_{5n+7}-(426+42\alpha)x_{5n+6}+5[(12\alpha+114)x_{3n+5}-(426+42\alpha)x_{3n+5}]+10[(12\alpha+114)x_{n+3}-(42+426\alpha)x_{n+2}]\right]$$
- $$\frac{1}{2(\alpha^2+10\alpha-2)}\left[(4\alpha+38)y_{5n+5}-18x_{5n+6}+5[(4\alpha+38)y_{3n+3}-18x_{3n+4}]+10[(4\alpha+38)y_{n+1}-18x_{n+2}]\right]$$
- $$\frac{1}{(\alpha^2+10\alpha-2)}\left[(4\alpha+38)y_{5n+6}-(66+6\alpha)x_{5n+6}+5[(4\alpha+38)y_{3n+4}-(66+6\alpha)x_{3n+4}]+10[(4\alpha+38)y_{n+2}-(66+6\alpha)x_{n+2}]\right]$$
- $$\frac{1}{2(\alpha^2+10\alpha-2)}\left[(4\alpha+38)y_{5n+7}-(246+24\alpha)x_{5n+6}+5[(4\alpha+38)y_{3n+5}-(246+24\alpha)x_{3n+4}]+10[(4\alpha+38)y_{n+3}-(246+24\alpha)x_{n+2}]\right]$$
- $$\frac{1}{7(\alpha^2+10\alpha-2)}\left[(14\alpha+142)y_{5n+5}-18x_{5n+7}+5[(14\alpha+142)y_{3n+3}-18x_{3n+5}]+10[(14\alpha+142)y_{n+1}-18x_{n+3}]\right]$$
- $$\frac{1}{2(\alpha^2+10\alpha-2)}\left[(14\alpha+142)y_{5n+6}-(66+6\alpha)x_{5n+7}+5[(14\alpha+142)y_{3n+4}-(66+6\alpha)x_{3n+5}]+10[(14\alpha+142)y_{n+2}-(66+6\alpha)x_{n+3}]\right]$$
- $$\frac{1}{(\alpha^2+10\alpha-2)}\left[(14\alpha+142)y_{5n+7}-(246+24\alpha)x_{5n+7}+5[(14\alpha+142)y_{3n+5}-(246+24\alpha)x_{3n+5}]+10[(14\alpha+142)y_{n+3}-(246+24\alpha)x_{n+3}]\right]$$
- $$\frac{1}{(\alpha^2+10\alpha-2)}\left[(2\alpha+22)y_{5n+5}-6y_{5n+6}+5[(2\alpha+22)y_{3n+3}-6y_{3n+4}]+10[(2\alpha+22)y_{n+1}-6y_{n+2}]\right]$$
- $$\frac{1}{4(\alpha^2+10\alpha-2)}\left[(8\alpha+82)y_{5n+5}-6y_{5n+7}+5[(8\alpha+82)y_{3n+3}-6y_{3n+5}]+10[(8\alpha+82)y_{n+1}-6y_{n+3}]\right]$$

- $\frac{1}{(\alpha^2+10\alpha-2)}[(8\alpha+82)y_{5n+6}-(22+2\alpha)y_{5n+7}+5[(8\alpha+82)y_{3n+4}-(22+2\alpha)y_{3n+5}]+10[(8\alpha+82)y_{n+2}-(22+2\alpha)y_{n+3}]]$

REMARKABLE OBSERVATIONS:

1. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in table :2 below

Table:2 Hyperbola

S.No	Hyperbola	(P,Q)
1.	$3P^2-Q^2=12(\alpha^2+10\alpha-2)^2$	$P=(2\alpha+10)x_{n+2}-(38+4\alpha)x_{n+1}$ $Q=(66+6\alpha)x_{n+1}-18x_{n+2}$
2.	$3P^2-Q^2=48(\alpha^2+10\alpha-2)^2$	$P=(\alpha+5)x_{n+3}-(71+7\alpha)x_{n+1}$ $Q=(123+12\alpha)x_{n+1}-9x_{n+3}$
3.	$3P^2-Q^2=12(\alpha^2+10\alpha-2)^2$	$P=(2\alpha+10)y_{n+1}-18x_{n+1}$ $Q=(30+6\alpha)x_{n+1}-18y_{n+1}$
4.	$3P^2-Q^2=12(\alpha^2+10\alpha-2)^2$	$P=(\alpha+5)y_{n+2}-(33+3\alpha)x_{n+1}$ $Q=(57+6\alpha)x_{n+1}-9y_{n+2}$
5.	$3P^2-Q^2=588(\alpha^2+10\alpha-2)^2$	$P=(2\alpha+10)y_{n+3}-(246+24\alpha)x_{n+1}$ $Q=(426+42\alpha)x_{n+1}-18y_{n+3}$
6.	$3P^2-Q^2=108(\alpha^2+10\alpha-2)^2$	$P=(12\alpha+114)x_{n+3}-(426+42\alpha)x_{n+2}$ $Q=(738+72\alpha)x_{n+2}-(18\alpha+198)x_{n+3}$
7.	$3P^2-Q^2=48(\alpha^2+10\alpha-2)^2$	$P=(4\alpha+38)y_{n+1}-18x_{n+2}$ $Q=(30+6\alpha)x_{n+2}-(6\alpha+66)y_{n+1}$
8.	$3P^2-Q^2=12(\alpha^2+10\alpha-2)^2$	$P=(4\alpha+38)y_{n+2}-(6\alpha+66)x_{n+2}$ $Q=(114+12\alpha)x_{n+2}-(66+6\alpha)y_{n+2}$
9.	$3P^2-Q^2=48(\alpha^2+10\alpha-2)^2$	$P=(4\alpha+38)y_{n+3}-(246+24\alpha)x_{n+2}$ $Q=(426+42\alpha)x_{n+2}-(6\alpha+66)y_{n+3}$
10.	$3P^2-Q^2=588(\alpha^2+10\alpha-2)^2$	$P=(14\alpha+142)y_{n+1}-18x_{n+3}$ $Q=(6\alpha+30)x_{n+3}-(24\alpha+246)y_{n+1}$

International Research Journal of Education and Technology

Peer Reviewed Journal

ISSN 2581-7795

11.	$3P^2 - Q^2 = 48(\alpha^2 + 10\alpha - 2)^2$	$P = (14\alpha + 142)y_{n+2} - (66 + 6\alpha)x_{n+3}$ $Q = (114 + 12\alpha)x_{n+3} - (24\alpha + 246)y_{n+2}$
12.	$3P^2 - Q^2 = 12(\alpha^2 + 10\alpha - 2)^2$	$P = (14\alpha + 142)y_{n+3} - (24\alpha + 246)yx_{n+3}$ $Q = (426 + 42\alpha)x_{n+3} - (24\alpha + 66)y_{n+3}$
13.	$3P^2 - Q^2 = 12(\alpha^2 + 10\alpha - 2)^2$	$P = (2\alpha + 22)y_{n+1} - 6y_{n+2}$ $Q = (10 + 2\alpha)y_{n+2} - (38 + 4\alpha)y_{n+1}$
14.	$3P^2 - Q^2 = 192(\alpha^2 + 10\alpha - 2)^2$	$P = (8\alpha + 82)y_{n+1} - 6y_{n+3}$ $Q = (10 + 2\alpha)y_{n+3} - (14\alpha + 142)y_{n+1}$
15.	$3P^2 - Q^2 = 12(\alpha^2 + 10\alpha - 2)^2$	$P = (8\alpha + 82)y_{n+2} - (22 + 2\alpha)y_{n+3}$ $Q = (38 + 4\alpha)y_{n+3} - (14\alpha + 142)y_{n+2}$

2.Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabola which are presented in the Table: 3 below:

Table: 3 Parabola

s.no	**Parabola**	**(R,Q)**
1	$3R(\alpha^2 + 10\alpha - 2) - Q^2 = 12(\alpha^2 + 10\alpha - 2)^2$	$R = (2\alpha + 10)x_{2n+3} - (4\alpha + 38)x_{2n+2} + 2(\alpha^2 + 10\alpha - 2)$ $Q = (66 + 6\alpha)x_{n+1} - 18x_{n+2}$
2	$6R(\alpha^2 + 10\alpha - 2) - Q^2 = 48(\alpha^2 + 10\alpha - 2)^2$	$R = (\alpha + 5)x_{4n+1} - (7\alpha + 71)x_{2n+2} + 4(\alpha^2 + 10\alpha - 2)$ $Q = (123 + 12\alpha)x_{n+1} - 9x_{n+3}$
3	$3R(\alpha^2 + 10\alpha - 2) - Q^2 = 12(\alpha^2 + 10\alpha - 2)^2$	$R = (2\alpha + 10)y_{2n+2} - 18x_{2n+2} + 2(\alpha^2 + 10\alpha - 2)$ $Q = (30 + 6\alpha)x_{n+1} - 18y_{n+1}$
4	$3R(\alpha^2 + 10\alpha - 2) - Q^2 = 12(\alpha^2 + 10\alpha - 2)^2$	$R = (\alpha + 5)y_{2n+3} - (3\alpha + 33)x_{2n+2} + 2(\alpha^2 + 10\alpha - 2)$ $Q = (57 + 6\alpha)x_{n+1} - 9y_{n+2}$
5	$21R(\alpha^2 + 10\alpha - 2) - Q^2 = 588(\alpha^2 + 10\alpha - 2)^2$	$R = (2\alpha + 10)y_{n+3} - (24\alpha + 246)x_{n+1} + 2(\alpha^2 + 10\alpha - 2)$ $Q = (426 + 42\alpha)x_{n+1} - 18y_{n+2}$
6	$9R(\alpha^2 + 10\alpha - 2) - Q^2 = 108(\alpha^2 + 10\alpha - 2)^2$	$R = [(114 + 12\alpha)x_{2n+4} - (426 + 42\alpha)x_{2n+3}] + 6(\alpha^2 + 10\alpha - 2)$ $Q = 6(738 + 72\alpha)x_{n+2} - (18\alpha + 198)x_{n+3}$
7	$6R(\alpha^2 + 10\alpha - 2) - Q^2 = 48(\alpha^2 + 10\alpha - 2)^2$	$R = [(38 + 4\alpha)y_{2n+2} - 18(x_{2n+3})] + 2(\alpha^2 + 10\alpha - 2)$ $Q = (36 + 6\alpha)x_{n+2} - (6\alpha + 66)y_{n+1}$
8	$3R(\alpha^2 + 10\alpha - 2) - Q^2 = 12(\alpha^2 + 10\alpha - 2)^2$	$R = [(38 + 4\alpha)y_{2n+3} - (6\alpha + 66)x_{2n+3}] + 2(\alpha^2 + 10\alpha - 2)$ $Q = (114 + 12\alpha)x_{n+2} - (66 + 6\alpha)y_{n+2}$
9	$6R(\alpha^2 + 10\alpha - 2) - Q^2 = 48(\alpha^2 + 10\alpha - 2)^2$	$R = (38 + 4\alpha)y_{2n+4} - (246 + 24\alpha)x_{2n+3} + 2(\alpha^2 + 10\alpha - 2)$ $Q = (426 + 42\alpha)x_{n+2} - (6\alpha + 66)y_{n+3}$

10	$21R(\alpha^2+10\alpha-2)-Q^2=588(\alpha^2+10\alpha-2)^2$	$R=(142+14\alpha)y_{2n+2}-18x_{2n+4}+14(\alpha^2+10\alpha-2)$ $Q=(6\alpha+30)x_{n+3}-(24\alpha+246)y_{n+1}$
11	$6R(\alpha^2+10\alpha-2)-Q^2=48(\alpha^2+10\alpha-2)^2$	$R=(142+14\alpha)y_{2n+3}-(66+6\alpha)x_{2n+4}+4(\alpha^2+10\alpha-2)$ $Q=(114+12\alpha)x_{n+3}-(24\alpha+246)y_{n+2}$
12	$3R(\alpha^2+10\alpha-2)-Q^2=12(\alpha^2+10\alpha-2)^2$	$R=(142+14\alpha)y_{2n+4}-(246+24\alpha)x_{2n+4}+2(\alpha^2+10\alpha-2)$ $Q=(426+42\alpha)x_{n+3}-(246+24\alpha)y_{n+3}$
13	$3R(\alpha^2+10\alpha-2)-Q^2=12(\alpha^2+10\alpha-2)^2$	$R=(2\alpha+22)y_{2n+2}-6y_{2n+3}+2(\alpha^2+10\alpha-2)$ $Q=(10+2\alpha)y_{n+2}-(38+4\alpha)y_{n+1}$
14	$12R(\alpha^2+10\alpha-2)-Q^2=192(\alpha^2+10\alpha-2)^2$	$R=(82+8\alpha)y_{2n+2}-6y_{2n+4}+8(\alpha^2+10\alpha-2)$ $Q=(10+2\alpha)y_{n+3}-(14\alpha+142)y_{n+1}$
15	$3R(\alpha^2+10\alpha-2)-Q^2=12(\alpha^2+10\alpha-2)^2$	$R=(82+8\alpha)y_{2n+3}-(2\alpha+22)y_{2n+4}+2(\alpha^2+10\alpha-2)$ $Q=(38+4\alpha)y_{n+3}-(14\alpha+142)y_{n+2}$

CONCLUSION*:*

In this paper, we have presented infinitely many integer solutions for the Diophantine equationsrepresentedby the positive pell equations $y^2=3x^2+\alpha^2+10\alpha-2$.As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of pell equations and determine the solutions with the suitable properties.

REFERENCES:

1. David M.Burton, Elementary Number Theory, Tata MC Graw hill publishing company, limited New Delhi-2000.
2. Mordel, L.J.,1969. Diophantine equation. New York: Academic Press.
3. Telang S.J., Number Theory, Tata McGraw Hill publishing Company limited New Delhi-2000.
4. S.Vidhyalakshmi, J.Shanthi, K.Hema, M.A.Gopalan " On the 2nd degree equation with three unknowns $5x^2+4y^2=189z^2$", International journal of Multidisciplinary Research and Growth Evalution,Volume 03, Issue 03,page no 197-204, Issue no:2582-7138 May-June 2022.
5. Dr. J.Shanthi,M.A.Gopalan,P.Dhanassree " Observation on the integral solutions of the Ternary quadratic equation $x^2+y^2=z^2+10$", International Research Journal of Education and Technology vol-04, Issue 07,Page no:220-231, Issue No:2581-7795,July 2022.
6. Dr. T. Mahalakshmi, P.Sowmiya" A Search on Integer solutions to the homogeneous quadratic equation with 3 unknows $x^2+14y^2=23z^2$", International Journal of Research Publication and Reviews. Vol-04, Issue 1, Page no:714-420, Issue No: 2582-7421 January 2023.
7. Dr. J.Shanthi, B.Indhumki, "Observation on the Hyperbola $x^2=20y^2-4$", International Journal of Research publication and Reviews. Vol-04, Issue 1, Page no:678-682, Issue No:2582-7421, January 2023.

8. Dr. J.Shanthi, M.Parkavi, "On finding Integer solutions to the Homogenous Ternary quadratic diophatineequation $2(x^2+y^2)-3xy=32z^2$" , International Journal of Research publication and reviews. Vol-04, Issue1, Page no:700-708, Issue no:2582-7421, January 2023.
9. Dr. J.Shanthi, M.Parkavi, "On finding integer Solutions to the Non-Homogeneous Ternary Quinticdiophatineequation $x^2+y^2-xy=28z^2$", International Research Journal of Education& Technology. Vol 5,Issue 03, Page no: 463-471, Issue No:2581-7795,March 2023.
10. S.Vidhyalakshmi, J.Shanthi, M.Devi,M.A.Gopalan "A study on the Hyperbola $y^2=87x^2+1$", International Research Journal of Education & Technology. Vol-05, Issue-03, Page.no:18-32, Issue No:2581-7795, March 2023.
11. Dr. J.Shanthi, M.Parkavi,"Observation on the Hyperbola $x^2=20y^2+45$". International Journal of Research publication and reviews. Vol-04,Issue 05, Page.no:570-583, Issue No:2582-7421,May 2023.
12. Dr. T.Mahalakshmi, E.Shalini, "On finding Integers Solutions to the Homogeneous Ternary quadratic Diophantine equations $3(x^2+y^2)-5xy=15z^2$ ",International Journal of Research publication and reviews. Vol-4,Issue 05, Page no:452-462, Issue No:2582-7421, May 2023.
13. Dr. J.Shanthi, T.Mahalakshmi, S.vidhyalakshmi, M.A.Gopalan" A search on Integer Solutions to the Homogeneous Quadratic Equation with three unknowns $x^2+17y^2=21z^2$", International Journal of Research publication and reviews. Vol-4, Issue 05, Page no:3610-3619, Issue No: 2582-7421, May 2023.